

Synthetic spin-orbit coupling in ultracold Λ -type atoms

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We consider the simulation of non-abelian gauge potentials in ultracold atom systems with atom-field interaction in the Λ configuration where two internal states of an atom are coupled to a third common one with a detuning. We find the simulated non-abelian gauge potentials can have the same structures as those simulated in the tripod configuration if we parameterize Rabi frequencies properly, which means we can design spin-orbit coupling simulation schemes based on those proposed in the tripod configuration. We show the simulated spin-orbit coupling in the Λ configuration can only be of a form similar to $p_x\sigma_y$ even when the Rabi frequencies are not much smaller than the detuning.

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I. INTRODUCTION

Many interesting quantum phenomena have been found in condensed matter physics when electrons have a spin-orbit coupling, such as the spin Hall effect and the topological insulator [1, 2]. Ultracold atom systems are now regarded as simulation platforms to study condensed matter physics [3], therefore it is important to realize spin-orbit couplings in these systems. It has been shown theoretically that abelian and non-abelian gauge potentials can be simulated for ultracold atoms via their interaction with laser fields, and non-abelian gauge potentials can be used to generate spin-orbit couplings [4–6]. A general spin-orbit coupling for ultracold atoms has not yet been realized experimentally, however some experimental progress towards this direction have been made [7–13].

Theoretical schemes to simulate spin-orbit couplings for ultracold atoms are usually proposed with atom-field interaction in the so called tripod configuration, where two dark states are used to form the effective spin space [14–20]. These schemes have a drawback that the two dark states are not the lowest energy dressed states, hence atom-atom interactions can induce collisional decay. Some authors also consider the simulation of spin-orbit coupling with atom-field interaction in the Λ configuration, where two lowest energy dressed states are used to form the effective spin space [21, 22]. Recently several experiments have realized the special spin-orbit coupling $p_x\sigma_y$ for ultracold atoms via Raman process [11–13], which is a scheme of Λ configuration with Rabi frequencies much smaller than the detuning. Some other kinds of methods are also proposed to simulate spin-orbit couplings in ultracold atom systems [23–25].

Although many interesting features of ultracold atoms have been theoretically found when they have a general spin-orbit coupling such as the Rashba and Dresselhaus spin-orbit couplings, currently we can only experimentally achieve the special spin-orbit coupling $p_x\sigma_y$ for ultracold atoms via Raman process. Since Raman process is a scheme of Λ configuration with Rabi frequencies much smaller than the detuning, it is natural to ask whether more general spin-orbit couplings can

be simulated in Λ configuration when Rabi frequencies are not much smaller than the detuning. We find the answer is NO at least in our concerned Λ configuration.

The structure of the paper is as follows. We first give an analytical expression of the simulated non-abelian gauge potentials in our concerned Λ configuration, which can help us to design spin-orbit coupling simulation schemes based on those proposed in the tripod configuration. We then consider a spin-orbit coupling simulation scheme where two plane waves are used in the Λ configuration. We find the simulated spin-orbit coupling can only be of a form similar to $p_x\sigma_y$ due to the non-degeneracy of the two lowest energy dressed states. We also analyze how the relative magnitude of the two lasers affect the simulated spin-orbit coupling Hamiltonian in this scheme.

II. NON-ABELIAN GAUGE POTENTIAL SIMULATION IN Λ CONFIGURATION

A general theory on the simulation of non-abelian gauge potentials for ultracold atoms is presented in Ref. [5]. Here we focus on atom-field interaction in the Λ configuration. As shown in FIG. 1, suppose two internal states $|1\rangle$ and $|2\rangle$ of an atom are coupled to a third common one $|3\rangle$ via laser fields with a detuning. The atom Hamiltonian will be

$$\hat{H} = \frac{\hat{p}^2}{2m} + \hat{H}_0 + V, \quad (1)$$

where \hat{H}_0 is the atom-field interaction and V is the possible external trapping potential. In the interaction picture,

$$\hat{H}_0 = \hbar\Delta|3\rangle\langle 3| + \hbar[\Omega_1|3\rangle\langle 1| + \Omega_2|3\rangle\langle 2| + H.c.], \quad (2)$$

where Δ is the detuning that is assumed to be positive, Ω_1 and Ω_2 are Rabi frequencies. We parameterize two Rabi frequencies as

$$\Omega_1 = \frac{\Delta}{2} \tan 2\theta \cos \phi e^{iS_1}, \Omega_2 = \frac{\Delta}{2} \tan 2\theta \sin \phi e^{iS_2}, \quad (3)$$

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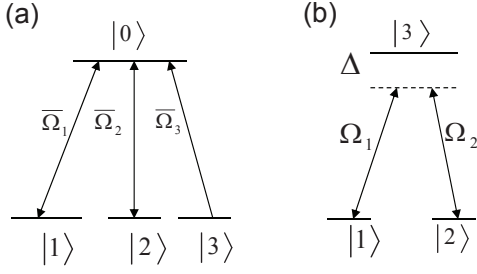


FIG. 1: (a) The tripod configuration and (b) Λ configuration. If in the tripod configuration Rabi frequencies are parameterized as $\bar{\Omega}_1 = \bar{\Omega} \sin \theta \cos \phi e^{iS_1}$, $\bar{\Omega}_2 = \bar{\Omega} \sin \theta \sin \phi e^{iS_2}$, and $\bar{\Omega}_3 = \bar{\Omega} \cos \theta e^{iS_3}$, and in the Λ configuration Rabi frequencies are parameterized as $\Omega_1 = \frac{\Delta}{2} \tan 2\theta \cos \phi e^{i(S_1-S_3)}$ and $\Omega_2 = \frac{\Delta}{2} \tan 2\theta \sin \phi e^{i(S_2-S_3)}$ with Δ being the detuning, then the simulated non-abelian gauge potentials in the Λ configuration have exactly the same structures as those simulated in the tripod configuration.

with $-\pi/4 < \theta < \pi/4$. Under this parameterization the three eigenstates of \hat{H}_0 are

$$\begin{aligned} |e_1\rangle &= \sin \phi e^{-iS_1} |1\rangle - \cos \phi e^{-iS_2} |2\rangle, \\ |e_2\rangle &= \cos \theta (\cos \phi e^{-iS_1} |1\rangle + \sin \phi e^{-iS_2} |2\rangle) - \sin \theta |3\rangle, \\ |e_3\rangle &= \sin \theta (\cos \phi e^{-iS_1} |1\rangle + \sin \phi e^{-iS_2} |2\rangle) + \cos \theta |3\rangle, \end{aligned} \quad (4)$$

with the corresponding eigenvalues being

$$E_1 = 0, E_2 = -\hbar\Delta \frac{\sin^2 \theta}{\cos(2\theta)}, E_3 = \hbar\Delta \frac{\cos^2 \theta}{\cos(2\theta)}. \quad (5)$$

The full quantum state of the atom can be written in the form $|\Phi\rangle = \sum_{i=1}^3 \Psi_i |e_i\rangle$. Due to the position dependence of the dressed states $|e_i\rangle$, when we substitute the full quantum state $|\Phi\rangle$ into the Schrödinger equation with \hat{H} given in Eq. (1), we can find that the column vector of wave functions $\Psi = (\Psi_1, \Psi_2, \Psi_3)^T$ satisfies the Schrödinger equation with Hamiltonian [5]

$$\tilde{H} = \frac{(\hat{\vec{p}} - \vec{A})^2}{2m} + \tilde{V}, \quad (6)$$

where \vec{A} and \tilde{V} are 3×3 matrices:

$$\vec{A}_{n,m} = i\hbar \langle e_n | \nabla | e_m \rangle, \tilde{V}_{n,m} = E_n \delta_{n,m} + \langle e_n | V | e_m \rangle. \quad (7)$$

We note that \vec{A} and V give contribution to the off-diagonal elements and E_n give contribution to the diagonal elements of \tilde{H} .

Since $\Delta > 0$ and $-\pi/4 < \theta < \pi/4$, there are $E_3 > E_1 \geq E_2$ and $E_3 - E_1 \geq \hbar\Delta$. Now we discuss the condition when the off-diagonal elements of \tilde{H} connecting Ψ_1 to Ψ_3 and Ψ_2 to Ψ_3 can be neglected, so that it can be used to simulate the movement of a particle with spin-1/2. Note that $\vec{A}_{n,m}$ usually has a magnitude of momentum P_L of the applied laser fields, therefore the off-diagonal elements of \tilde{H} connecting Ψ_1 to Ψ_3 and Ψ_2 to Ψ_3 can be neglected when

$\frac{P_L^2}{2m} \ll \hbar\Delta$, $|\langle e_n | V | e_m \rangle| \ll \hbar\Delta$ and atoms move very slowly (i.e., $\frac{(\vec{p})^2}{2m} \ll \hbar\Delta$). If these conditions are satisfied the wave functions $(\Psi_1, \Psi_2)^T$ will be approximately decoupled from Ψ_3 and evolve under the Hamiltonian [5]

$$\hat{H}_{eff} = \frac{(\hat{\vec{p}} - \vec{A})^2}{2m} + \tilde{V} + \Phi. \quad (8)$$

Here \vec{A} , \tilde{V} and Φ are 2×2 matrices, the elements of \vec{A} and \tilde{V} are described in Eq. (7) with $n, m = 1, 2$, and

$$\Phi_{n,m} = \frac{1}{2m} \vec{A}_{n,3} \cdot \vec{A}_{3,m}, \quad n, m = 1, 2. \quad (9)$$

The Hamiltonian \hat{H}_{eff} in Eq. (8) simulates the movement of a particle with spin-1/2 in gauge potentials, where two lowest energy dressed states $|e_1\rangle$ and $|e_2\rangle$ represent spin up and spin down respectively. Here we emphasize that the θ is not required to be small to get \hat{H}_{eff} , i.e., the magnitudes of Rabi frequencies Ω_1 and Ω_2 are not required to be much smaller than the detuning Δ .

We can get an analytical expression of the simulated gauge potentials \vec{A} and Φ in \hat{H}_{eff} by substituting Eq. (4) into Eq. (7). But note that the dressed states $|e_1\rangle$, $|e_2\rangle$ and $|e_3\rangle$ have the same mathematical structures as the two dark states $|D_1\rangle$, $|D_2\rangle$ [5] and the bright state $|B\rangle = |D_0\rangle$ [26] in the tripod configuration respectively, and in the tripod configuration the simulated gauge potentials can be expressed as $\vec{A}_{n,m} = i\hbar \langle D_n | \nabla | D_m \rangle$ and $\Phi_{n,m} = \frac{1}{2m} \vec{A}_{n,0} \cdot \vec{A}_{0,m}$ [26], we can conclude immediately that the simulated gauge potentials in our concerned Λ configuration have the same mathematical structures as those simulated in the tripod configuration, i.e., we can obtain an analytical expression for the simulated gauge potentials \vec{A} and Φ of \hat{H}_{eff} in our concerned Λ configuration just through replacing S_{13} and S_{23} in Eq. (13) and Eq. (14) of Ref. [5] by S_1 and S_2 respectively. Here we write them down for completeness:

$$\begin{aligned} \vec{A}_{1,1} &= \hbar(\cos^2 \phi \nabla S_2 + \sin^2 \phi \nabla S_1), \\ \vec{A}_{1,2} &= \hbar \cos \theta \left[\frac{1}{2} \sin(2\phi) (\nabla S_1 - \nabla S_2) - i \nabla \phi \right], \\ \vec{A}_{2,2} &= \hbar \cos^2 \theta (\cos^2 \phi \nabla S_1 + \sin^2 \phi \nabla S_2), \\ \Phi_{1,1} &= \frac{\hbar^2}{2m} \sin^2 \theta \left[\frac{1}{4} \sin^2(2\phi) (\nabla S_1 - \nabla S_2)^2 + (\nabla \phi)^2 \right], \\ \Phi_{1,2} &= \frac{\hbar^2}{2m} \sin \theta \left[\frac{1}{2} \sin(2\phi) (\nabla S_1 - \nabla S_2) - i \nabla \phi \right] \\ &\quad \cdot \left[\frac{1}{2} \sin(2\theta) (\cos^2 \phi \nabla S_1 + \sin^2 \phi \nabla S_2) - i \nabla \theta \right], \\ \Phi_{2,2} &= \frac{\hbar^2}{2m} \left[\frac{1}{4} \sin^2(2\theta) (\cos^2 \phi \nabla S_1 + \sin^2 \phi \nabla S_2)^2 + (\nabla \theta)^2 \right]. \end{aligned} \quad (10)$$

We will give explicit examples of S_1 and S_2 to show how spin-orbit coupling Hamiltonian can be obtained.

Our parameterization of the two Rabi frequencies not only gives us a convenient way to obtain an analytical expression of the simulated gauge potentials as shown above, but also gives us a way to design spin-orbit simulation schemes in the

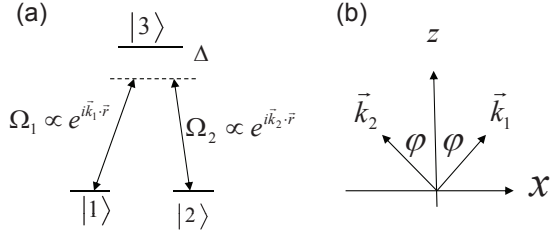


FIG. 2: (a) Two plane waves coupling two lower internal states of an atom to a common higher internal states. (b) The directions of the two plane waves.

A configuration based on those proposed in the tripod configuration. Suppose there is a scheme to simulate spin-orbit couplings in the tripod configuration with the Rabi frequencies $\Omega_1 = \bar{\Omega} \sin \theta \cos \phi e^{iS_1}$, $\Omega_2 = \bar{\Omega} \sin \theta \sin \phi e^{iS_2}$ and $\Omega_3 = \bar{\Omega} \cos \theta e^{iS_3}$, then in our concerned Λ configuration we can design a counterpart spin-orbit coupling simulation scheme using the Rabi frequencies $\Omega_1 = \frac{\Delta}{2} \tan 2\theta \cos \phi e^{i(S_1 - S_3)}$ and $\Omega_2 = \frac{\Delta}{2} \tan 2\theta \sin \phi e^{i(S_2 - S_3)}$ with Δ being the detuning. These two schemes simulate the same kind of spin-orbit coupling since the gauge potentials simulated by them have exactly the same mathematical structures, where the difference is that there are some restrictions on the parameter θ and there is a Zeeman term in \hat{H}_{eff} due to the non-degeneracy of the two lowest energy dressed states in the Λ configuration. The known schemes to simulate spin-orbit couplings of Refs. [21, 22] in the Λ configuration can be regarded as the counterpart schemes of Refs. [14, 15] in the tripod configuration where standing waves are used. In the next section we will consider a new spin-orbit coupling simulation scheme where only two plane waves are used, which can be regarded as the counterpart scheme of Refs. [16–20] in the tripod configuration where only plane waves are used.

III. SPIN-ORBIT COUPLING SIMULATION IN Λ CONFIGURATION

In this section we consider a spin-orbit coupling simulation scheme where two plane waves are used in the Λ configuration. This is an example to show the simulated spin-orbit coupling in the Λ configuration can only be of a form similar to $p_x \sigma_y$ even when the Rabi frequencies are not much smaller than the detuning. We also analyze how the relative magnitude of the two lasers affect the form of the simulated spin-orbit coupling Hamiltonian in this scheme.

As shown in FIG. 2 two internal states $|1\rangle$ and $|2\rangle$ of an atom are coupled to a third common one $|3\rangle$ via two plane waves respectively with the same detuning. The directions of the two waves are in the x - z plane, and the phases of the two Rabi frequencies are assumed to be

$$\begin{aligned} S_1 &= \vec{k}_1 \cdot \vec{r} = kx \sin \phi + kz \cos \phi, \\ S_2 &= \vec{k}_2 \cdot \vec{r} = -kx \sin \phi + kz \cos \phi, \end{aligned} \quad (11)$$

where k is the wave vector of the applied plane waves and ϕ determines the wave directions. Substitute S_1 and S_2 into the expressions of simulated gauge potentials \vec{A} and Φ in Eq. (10) and note that only S_1 and S_2 are position dependent, we get $A_y = 0$ and

$$\begin{aligned} \frac{A_x}{\hbar k} &= -\frac{1}{2} \sin^2 \theta \cos(2\phi) \sin \phi \sigma_0 + \cos \theta \sin(2\phi) \sin \phi \sigma_x \\ &\quad - \frac{1}{2} (2 - \sin^2 \theta) \cos(2\phi) \sin \phi \sigma_z, \\ \frac{A_z}{\hbar k} &= \frac{1}{2} (2 - \sin^2 \theta) \cos \phi \sigma_0 + \frac{1}{2} \sin^2 \theta \cos \phi \sigma_z, \end{aligned} \quad (12)$$

where σ_x , σ_y , σ_z are three Pauli matrices and σ_0 is the 2×2 identity matrix. If we set $\phi = \pi/4$ and substitute \vec{A} above into \hat{H}_{eff} then we can get a spin-orbit coupling proportional to $\cos \theta \sin \phi p_x \sigma_x + \frac{1}{2} \sin^2 \theta \cos \phi p_z \sigma_z$ which will be proportional $p_x \sigma_x + p_z \sigma_z$ when we choose proper θ and ϕ and can be further turned into Rashba or Dresselhaus spin-orbit coupling through changing spin basis. However it will be a different story when the Zeeman term in \hat{H}_{eff} due to the non-degeneracy of the two lowest energy dressed states is considered.

For simplicity we assume the external trapping potential $V = 0$, so that the Zeeman term \tilde{V} in the simulated Hamiltonian \hat{H}_{eff} in Eq. (8) will be

$$\tilde{V} = -\hbar \Delta \frac{\sin^2 \theta}{2 \cos(2\theta)} \sigma_0 + \hbar \Delta \frac{\sin^2 \theta}{2 \cos(2\theta)} \sigma_z. \quad (13)$$

Recall the effective Hamiltonian \hat{H}_{eff} is simulated under the conditions $\frac{(\hbar k)^2}{2m} \ll \hbar \Delta$ and $\frac{(\vec{p})^2}{2m} \ll \hbar \Delta$. Now we show under these conditions some terms in \hat{H}_{eff} can be further neglected. First we find the potential Φ satisfies

$$|\Phi_{n,m}| \leq \frac{(\hbar k)^2}{2m} \sin^2 \theta \ll \hbar \Delta \frac{\sin^2 \theta}{\cos(2\theta)}, \quad (14)$$

therefore Φ can be ignored compared to \tilde{V} . Second we write $\cos \theta$ in A_x of Eq. (12) as $1 - 2\sin^2(\theta/2)$ and then substitute A_x and A_z into H_{eff} , we can get some energy terms proportional to $\sin^2 \theta$ or $2\sin^2(\theta/2)$ [27], which are of magnitude $\max[\frac{(\vec{p})^2}{2m}, \frac{(\hbar k)^2}{2m}]$ and can also be ignored compared to \tilde{V} . Therefore in \hat{H}_{eff} we can safely write $\Phi = 0$ and

$$\begin{aligned} A_x &= \hbar k \sin \phi [\sin(2\phi) \sigma_x - \cos(2\phi) \sigma_z], \\ A_z &= \hbar k \cos \phi \sigma_0. \end{aligned} \quad (15)$$

We note that this result is obtained due to the non-degeneracy of the dressed states $|e_1\rangle$ and $|e_2\rangle$, not by assuming θ is close to zero. From Eq. (15) we find that only the movement in the x direction of the atom is coupled to its pseudo spin, and the coupled movement is governed by

$$\begin{aligned} \hat{H}_{xs} &= \frac{(\hat{p}_x - A_x)^2}{2m} + \tilde{V} \\ &= \frac{\hat{p}_x^2}{2m} + 2\alpha \hat{p}_x [\cos(2\phi) \sigma_z - \sin(2\phi) \sigma_x] + \hbar \sigma_z, \end{aligned} \quad (16)$$

where $\alpha = \frac{1}{2m}\hbar k \sin\varphi$, $h = \hbar\Delta \frac{\sin^2\theta}{2\cos(2\theta)}$ and a constant term $c = \frac{(\hbar k)^2}{2m}\sin^2\varphi - \hbar\Delta \frac{\sin^2\theta}{2\cos(2\theta)}$ is neglected. If we use $|e'_1\rangle = \cos\phi|e_1\rangle - \sin\phi|e_2\rangle$ and $|e'_2\rangle = \sin\phi|e_1\rangle + \cos\phi|e_2\rangle$ instead of $|e_1\rangle$ and $|e_2\rangle$ to represent spin up and spin down respectively, then the coupled Hamiltonian will be

$$\hat{H}'_{xs} = \frac{\hat{p}_x^2}{2m} + 2\alpha\hat{p}_x\sigma_z + h[\cos(2\phi)\sigma_z + \sin(2\phi)\sigma_x], \quad (17)$$

where $2\alpha\hat{p}_x\sigma_z$ represents spin-orbit coupling. Thus we have shown the simulated spin-orbit coupling in our concerned Λ configuration can only be of a form similar to $p_x\sigma_y$ even when the Rabi frequencies are not much smaller than the detuning.

The roles of h and ϕ seem clear in \hat{H}'_{xs} ; one controls the magnitude of the "magnetic field" and the other controls its direction. However, as we change ϕ , which is determined by the relative magnitude of two lasers, not only \hat{H}'_{xs} but also the spin basis states $|e'_1\rangle$ and $|e'_2\rangle$ will be changed, and it may be not easy to see the underlying physics. Now we assume $\hbar\Delta$ is so bigger that a small $\theta = \theta_m$ can lead to $h\sigma_z$ dominating \hat{H}_{xs} , then we can study the underlying physics only varying θ between $-\theta_m$ and θ_m . Since θ is small, i.e., the Rabi frequencies are much smaller than the detuning, there is $|e_2\rangle = \cos\phi e^{-iS_1}|1\rangle + \sin\phi e^{-iS_2}|2\rangle$ and our simulation scheme reduces to the Raman process in recent experiment [11]. At this time we get $|e'_1\rangle = -e^{-iS_2}|2\rangle$ and $|e'_2\rangle = e^{-iS_1}|1\rangle$, which are independent of ϕ . Experimentally we can study the phase transition of \hat{H}'_{xs} due to the change of ϕ and θ as in Refs. [11, 12].

IV. DISCUSSION AND SUMMARY

We have given an example that the simulated spin-orbit coupling in the Λ configuration can only be of a form sim-

ilar to $p_x\sigma_y$ even when the Rabi frequencies are not much smaller than the detuning. The same conclusion can also be obtained when we consider the spin-orbit coupling simulation schemes in Refs. [21, 22]. So if we want to get a more general spin-orbit coupling in our concerned Λ configuration, we should find a way to eliminate the Zeeman term in \hat{H}_{eff} due to the non-degeneracy of the two lowest energy dressed states. If we assume the trapping potential $V = V_1|1\rangle\langle 1| + V_2|2\rangle\langle 2| + V_3|3\rangle\langle 3|$ with $V_1 = V_2$ and $V_3 = V_1 - E_2/\sin^2\theta$, then this Zeeman term will be eliminated. However this trapping potential will give a coupling between Ψ_2 and Ψ_3 that cannot be ignored. How to effectively eliminate the Zeeman term due to the non-degeneracy of the two lowest energy dressed states is still under investigation.

In summary, we have given an analytical expression of the simulated non-abelian gauge potentials in our concerned Λ configuration based on a special parameterization of the two Rabi frequencies. We have shown the simulated spin-orbit coupling in our concerned Λ configuration can only be of a form similar to $p_x\sigma_y$ even when the Rabi frequencies are not much smaller than the detuning.

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